Department of Mathematics & Philosophy of Engineering

Faculty of Engineering Technology

The Open University of Sri Lanka

Nawala - Nugegoda

Course: MPZ3132 – Engineering Mathematics IB

Academic Year – 2013/2014

Instructions

- **Answer all the questions in each assignment.**
- **In these assignments a , b and c are <u>non – zero, distinct and three digits from theextreme right of your registration number</u>. See the examples**
  - **If the registration number is 30456601 then, since from the extreme right the first digit is 1, therefore $a = 1$. Since the second digit is zero and the third digit is 6 then $b = 6$. Since the fourth digit is also 6 and as a , b,and c are distinct then $c = 5$.**
  - **If the registration number is 1036300021 then $a = 1$, $b = 2$ and $c = 3$**
- **Write your address on the back page of your answer script.**
- **Use both sides of the papers to answer the assignments.**
- **Please send the answer scripts of your assignment on or before the due date to the following address.**

Course Coordinator – MPZ 3132

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You can collect model answers from the virtual class (www.ou.ac.lk)

User name – student0   Password – MPZ3132

Assignment No.01

Rewrite all the questions substituting your values of a, $b$and c (Five marks from100)

1. State the Dirichlet's conditions and write down the Fourier series expansion of a period function $f(x)$.

The function $f(x)$ is defined so that $f(x) = \begin{cases} 2ax + 3bx^2 + 4cx^3 \text{ for } -\pi < x < \pi \\ f(x + 2\pi) \text{ for } x \leq -\pi \text{ or } x \geq \pi \end{cases}$.

Find the Fourier series expansion of $f(x)$ Hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

2. The function$f(x)$ is defined such that $f(x) = ax^2 + b$ where $0 < x < 1$.

(a) Extend the function $f(x)$

(i) As an odd periodic function with period 2.

(ii) As an even periodic function with period 2.

(iii) As a periodic function with period 1.

(b) Draw the graphs of the above three functions on$[-4 , 4]$.

(c) Find the Fourier series expansion of $f(x)$

(i) As a sine series with period 2.

(ii) As a cosine series with period 2.

(iii) As a full trigonometric series with period 1.

3. $f(x)$ is defined as $f(x) = x(\pi - x)$ where $0 < x < \pi$.

(a) Using half range methods Prove that

$$f(x) = \frac{\pi^2}{6} - \sum_{n=1}^{\infty} \frac{\cos(2nx)}{n^2} = \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)\,x}{(2n-1)^2}.$$

(b) Using the Parseval's formula deduce that

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90} \text{ and } \sum_{n=1}^{\infty} \frac{1}{n^6} = \frac{\pi^4}{945}$$

4. Define the 'Taylor polynomial'.

(a) prove that

$$(a^2x^2+b^2)\frac{d^2y}{dx^2}+2a^2x\frac{dy}{dx}=0 \text{ If } y=tan^{-1}\left(\frac{ax}{b}\right).$$

Find the values of $\left(\frac{d^ny}{dx^n}\right)_{x=0}$ for $n\ = 1, 2, 3, 4, 5.$

Hence, deduce the order five 'Taylor polynomial for $y = tan^{-1}\left(\frac{ax}{b}\right).$

(b) If $y = \ln(a + bsinx)$ prove that

(**i**) $e^y\frac{dy}{dx}=bcosx$

(**ii**) $e^y\frac{d^2y}{dx^2}+e^y\left(\frac{dy}{dx}\right)^2 = -bsinx$

(**iii**) $\frac{d^3y}{dx^3}+3\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^3+\left(\frac{dy}{dx}\right)=0$

(**iv**) Find the values of $\left(\frac{d^ny}{dx^n}\right)_{x=0}$ for $n = 1, 2, 3, 4, 5.$

(**v**) Hence, deduce the order five Taylor polynomial for $y = \ln(a + bsinx)$

5. Write down the Taylor series expansion of $f(x)$ about $x = 0$.

Given that $f(x) = ln(b + ax)$ where $|x| < \frac{b}{a}$

(a) By using the principle of mathematical induction prove that

$$f(x) = \frac{(n-1)!\,(-1)^{n-1}a^n}{(b+ax)^n} \quad \text{where } n \geq 1.$$

(b) Find the 'Taylor' series expansion of $f(x) = ln(b + ax)$ about $x = 0.$

(c) Deduce the Taylor series expansions of

$$ln\left|\frac{b+ax}{b-ax}\right| \text{ and a infinte series for } ln3.$$

(d) Deduce the Taylor series expansions of $ln|b^2 - abx - 2a^2x^2|$ and the radius of convergence.

End

Assignment No.02

Rewrite all the questions substituting your values of a, $b$and c(Five marks from100)

1. If y_1 and y_2 are particular integrals of the differential equations
$\frac{d^2y}{dx^2} + \alpha\frac{dy}{dx} + \beta y = f_1(x)$ and $\frac{d^2y}{dx^2} + \alpha\frac{dy}{dx} + \beta y = f_2(x)$ respectively
prove that $Ay_1 + By_2$ is a particular integral of the differential equation
$\frac{d^2y}{dx^2} + \alpha\frac{dy}{dx} + \beta y = Af_1(x) + Bf_2(x)$. Where A and B areconstants.

(a) The differential equation
$\frac{d^2y}{dx^2} + 2a\frac{dy}{dx} + (a^2 + b^2)y = a + bcos(cx)$ has a trial function of the form
$y_T = l + mcos(cx) + nsin(cx)$ where l, m and n are constants.

(i) Find the values of l, m and n .

(ii) Find the general solution of the above differential equation.

(b) Using a suitable trial function

(i) find a particular integral for the differential equation

$$\frac{d^2y}{dx^2} + 2a\frac{dy}{dx} + (a^2 + b^2)y = x^2e^{cx}.$$

(ii) Find the general solution of the above differential equation.

(c) Deduce the general solution of the following differential equation

$$\frac{d^2y}{dx^2} + 2a\frac{dy}{dx} + (a^2 + b^2)y = 2013\big(a + bcos(cx)\big) + 2014x^2e^{cx}.$$

2. Prove that $\frac{1}{D+\alpha}f(x) = e^{-\alpha x}\frac{1}{D}e^{\alpha x}f(x)$.

(a) Consider the following system of differential equations

$$\frac{dx_1}{dt} = ax_1 + bx_2 \text{ and } \frac{dx_2}{dt} = bx_1 + ax_2.$$

(i) Find the solutions of the characteristic equation of the above system.

(ii) Find the general solutions of the above system.

(b) Find the particular solutions for the following system differential equations

$$\frac{dx_1}{dt} = ax_1 + bx_2 + e^{at} \text{ and } \frac{dx_2}{dt} = bx_1 + ax_2 + e^{-bt}.$$

(c) Hence, find the general solutions of the above system.

3. Write down the Laplace transformation of a function.

(a) Prove that

$$(\boldsymbol{i})\quad L(e^{\alpha t}) = \frac{1}{s-\alpha}$$

$$(\mathbf{ii})\quad L(\cosh(\beta t)) = \frac{s}{s^2-\beta^2}$$

$$(\mathbf{iii})\quad L(\sinh(\beta t)) = \frac{\alpha}{s^2-\beta^2}$$

(b) Using that shift theorem, deduce that

$$(\mathbf{i})\quad L(e^{\alpha t}\cosh(\beta t)) = \frac{s-\alpha}{(s-\alpha)^2-\beta^2}$$

$$(\mathbf{ii})\quad L(e^{\alpha t}\sinh(\beta t)) = \frac{\beta}{(s-\alpha)^2-\beta^2}$$

(c) Using the Laplace transformation solve the following system of equations under the initial condition $x_1(0) = l \; and \; x_2(0) = m$

$$\frac{dx_1}{dt} = ax_1 + bx_2 \text{ and } \frac{dx_2}{dt} = bx_1 + ax_2$$

4. Write down the Laplace transformation of function.

(a) Prove that

$$(\mathbf{i})\quad L(\cos(\beta t)) = \frac{s}{s^2+\beta^2}$$

$$(\mathbf{ii})\quad L(\sin(\beta t)) = \frac{\alpha}{s^2+\beta^2}$$

(b) Using that shift theorem, deduce that

$$(\boldsymbol{i})\quad (e^{\alpha t}\cos(\beta t)) = \frac{s-\alpha}{(s-\alpha)^2+\beta^2}$$

$$(\boldsymbol{ii})\quad L(e^{\alpha t}\sin(\beta t)) = \frac{\beta}{(s-\alpha)^2+\beta^2}$$

(c) Using $L(t^n f(t)) = (-1)^n \dfrac{d^n F(s)}{ds^n}$ prove that

$$L(te^{\alpha t}\sin(\beta t)) = \frac{2\beta}{(s-\alpha)^2+\beta^2} - \frac{2\beta^3}{[(s-\alpha)^2+\beta^2]^2}$$

(d) Using the Laplace transformation find the solution of the of differential equation

$$\frac{d^2y}{dx^2} + 2a\frac{dy}{dx} + (a^2+b^2)y = e^{ax} sinbx \text{ where } y'(0) = 1 \;\&\; y(0) = 0$$

End

Assignment No.03

Rewrite all the questions substituting your values of a, $b$and c (Five marks from100)

1. Let Z be any complex number. Define $argZ$.

(**a**) Draw the curves of $Re(aZ) = |aZ - b|$ and $|Z| = \sqrt{2}\frac{b}{a}$ in the same diagram.

(b) Find the complex numbers of the intersection points of the above curves.

(c) Shade the regions satisfying the following inequalities

$Re(aZ) \geq |aZ - b|$ and $|Z| \leq \sqrt{2}\frac{b}{a}$.

2. Let Z be any complex number. Define $LogZ$.

(**a**) Prove that $tan\left(ilog\left(\frac{a+ib}{a-ib}\right)\right) = \frac{2ab}{b^2 - a^2}$.

(b) Find the values of $Log(a - ib)$.

(**c**) Prove that $Log(1 + icot\theta) = \ln(cosec\theta) + i\left(2k\pi + \frac{\pi}{2} - \theta\right)$

(**d**)The function f is defined as follows $f(z) = \frac{z^2}{a+ib}$ where $|z| = c$ and $0 < \arg(z) \leq \frac{\pi}{2}$

(i) Determine the image of the above function.

(ii) Draw the domain and image of the above function.

3. Write down the definition of $\lim_{Z\to z_0} f(Z)$.

(**a**) If $\frac{1}{(Z^2 + a^2)(Z + b))} = \frac{A}{Z + \gamma} + \frac{B}{Z + \beta} + \frac{C}{Z + b}$ find the values of A, B, C, β and γ.

(b) Evaluate the following limits

(**i**) $\lim_{Z\to(2+i)b}\left(\frac{a^2Z^2 - 4abZ + 5b^2}{Z - (2 + i)b}\right)$

(**ii**) $\lim_{Z\to -\frac{ib}{a}}\left(\frac{a^3Z^3 - ib^3}{aZ + ib}\right)$

(**c**) If $f(Z) = \frac{aZ - b}{bZ + a}$ where $Z \neq -\frac{a}{b}$

(i) Show that f is one to one and on to.

(ii) Find the inverse function of the above $f(Z)$.

4. Prove that the moment generating function of the normal distribution which has the probability density function $f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$ where $x \in R$ is $M(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$.

(a) Hence find the mean and variance of the normal distribution.

(b) The masses of mangos are normally distributed. 5% of the mangos have a mass greater than $85g$ and 10% have a mass less than $25g$.

(c) Find the mean and the variance of the distribution.

(d) Calculate the symmetrical limits about the mean, within which 75% of the masses lie.

5. Prove that the moments generating function of Poisson distribution with parameter λ is $e^{\lambda(e^t-1)}$

(a) Hence find the mean and the variance of the Poisson distribution.

(b) The mean number of bacteria per millilitre of a liquid is known to be 4. Assuming that the number of bacteria follows a Poisson distribution,

(i) Find the probability that, in one millilitre of liquid there will be

A no bacteria

B 4 bacteria

C less than three bacteria

(ii) Find the probability

A in 3 ml of liquid there will be less than 2 bacteria

B in $\frac{1}{2}$ ml of liquid then will be more than 2 bacteria

End

Assignment No.04

Rewrite all the questions substituting your values of a, $b$and c (Five marks from100)

1. **(a)** Find the moments of inertia of the following bodies about the given axis.

(i) A uniform circular ring mass m and radius r, the axis perpendicular to the plane of the ring passing through the center.

(ii) A uniform circular disc of radius r, the axis perpendicular to the plane of the disc passing through the center.

(iii) A uniform solid cone of radius r height h and vertex A, the axis of the cone.

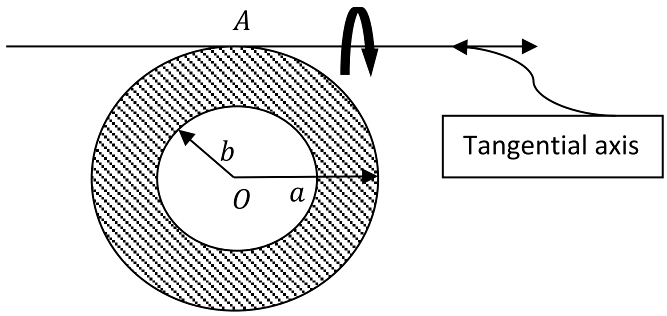
(b). Using suitable theorems,

(i) Deduce the moments of inertia of the above disc about the axis perpendicular to the plane of the disc passing through a point of its edge.

(ii) Deduce the moment of inertia of a uniform solid cone about the line perpendicular to the axis of the cone.

2. **(a)** Find the moments of inertia of a uniform circular ring of radius a and mass m about the perpendicular axis to the plane of the ring passing through the center.

(b) A uniform lamina in the form of circular annulus center O, mass M, outer radius a and inner radius b is shown below. Find the moments of inertia of the lamina about an axis through O perpendicular to the plane of the lamina.



(i) Find the moments of inertia of the lamina about its diameter.

(ii) Find the moments of inertia of the lamina about the tangential axis. (Refer the figure).

(iii) Now the circular annulus is free to rotate about the tangential axis. When the circular annulus is vertical it is projected with an angular velocity ω.

A. Find the angular velocity after the circular annulus turns at angle θ.

B. Find the period for the small oscillation of the circular annulus.

3. A particle is projected upwards with speed u from a point O along a greatest slope line of an inclined plane at an angle $sin^{-1}\left(\frac{1}{n^2}\right)$ (where $n > 1$) to the horizontal. The resistance to the motion on a unit mass is g$\left(\frac{b^2v^4}{a^2u^4}\right)$ where v is the velocity of the particle at a distance x from O.

(a) Prove that

$$\frac{d}{dx}(v^2) = -2g\left(\frac{a^2u^4 + n^2b^2v^4}{b^2n^2u^4}\right)$$

(b) Hence deduce the maximum distance traveled by the particle in the upward direction.

(c) Find the speed of the particle when it returns to O with the above resistance.

4. A particle P moves on a plane. At time t the polar coordinates of P is (r, θ) and the position vector of P is $\boldsymbol{r}$. If the unit vector along $\boldsymbol{r}$ is $\boldsymbol{l}$ and the unit vector perpendicular to $\boldsymbol{r}$ and θ increasing direction is $\boldsymbol{m}$.

(a) Prove that the velocity $\boldsymbol{v} = \dot{r}\boldsymbol{l} + r\dot{\theta}\boldsymbol{m}$ & the acceleration

$$\boldsymbol{f} = (\ddot{r} - r\ddot{\theta})\boldsymbol{l} + \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\boldsymbol{m}$$

(b) A particle of mass m on a smooth table is attached by a string passing through a small hole in the table and carries an equal particle hanging vertically. The first particle is projected with velocity $\sqrt{2gh}$ along the table at right angles to the string, when at a distance$l$ from the hole. If r is the distance from the hole at time t, prove the results

(i) $\left(\frac{dr}{dt}\right)^2 = gh\left(1 - \frac{l^2}{r^2}\right) + g(l - r)$

(ii) The lower particle will be pulled up to the hole if the total length of the string is less than $l + \frac{h}{2} + \sqrt{lh + \frac{h^2}{4}}$

(iii) The tension of the string is $2mg\left(1 + \frac{2l^2h}{r^3}\right)$.

5. A uniform rectangular lamina $ABCD$ is immersed vertically in a homogeneous liquid such that AB and CD are horizontal. The distances to AB and CD from the surface of the liquid are h_1 and h_2 respectively. Prove that that the centre of pressure of the lamina is at the distance $\frac{2}{3}\left(\frac{h_1^2 + h_1 h_2 + h_2^2}{h_1 + h_2}\right)$ from the free liquid surface.

(a) A square lamina $ABCD$ of side d is immersed in a homogeneous liquid of density ρ with its plane vertical and AB in the free surface. Find the liquid thrust on the lamina and the centre of pressure.

(b) A square $A'B'C'D'$ of side $\lambda d(\lambda < 1)$ is cut in the lamina. $ABCD$ and $A'B'C'D'$ have the same centre and their corresponding side are parallel. Find the thrust on the perforated lamina and show that the centre of pressure lies in $C'D'$ if $\lambda = \frac{3-\sqrt{5}}{2}$.

End